GS02 1052 – Imaging Science

Course description: This course provides a concise and coherent review of some commonly-encountered topics in applied mathematics, with a particular emphasis on their applications and relevance to imaging science. The course covers and is equally divided into two major sections: optimization methods and algorithms and Fourier and wavelet transforms.

Learning objectives:

- Develop a mathematical formalism for approaching some commonly encountered problems in medical physics
- Understand common computational techniques for solving optimization problems
- Understand the mathematical formulation, properties, and limitations of Fourier transforms (continuous and discrete) and wavelet transforms and how they relate to some medical physics problems

Time and location: 11:00am-12:00pm (Monday, Friday)
FCT14.5059

Syllabus:

Class	Instructor	Topic
1 (8/27)	Fuentes	1. Preliminaries - Algorithm Complexity, BLAS
2 (8/31)	Fuentes	2. Vector and metric spaces
3 (9/7)	Fuentes	3. Entropy, Mutual Information
4 (9/10)	Fuentes	4. Entropy, Mutual Information
5 (9/14)	Fuentes	5. Linear Independence, Equivalence of Norms
6 (9/17)	Fuentes	6. Linear Operators, Null space, Range Space
7 (9/21)	Fuentes	7. Point Spread Function, Operator Inverse
8 (9/24)	Fuentes	8. Rank and Nullity, Bounded Operator, Stability of Linear Systems
9 (9/28)	Fuentes	9. Inner Product, Orthogonality, Eigen-formulation
10 (10/1)	Fuentes	10. Spectral Methods, Low Rank Decomposition
11 (10/5)	Fuentes	11. Optimization Characterization of Solution
12 (10/8)	Fuentes	12. Line search, Newton-CG Trust-Region Methods
13 (10/12)	Fuentes	13. Least Square, QR Decomposition
14 (10/15)	Fuentes	14. Applications: L1 minimization
15 (10/19)	Ma	1. Where Nature does or calls for FT, definition of FT, notations, symmetry and FT, interpretation of FT
16 (10/22)	Ma	2. Some useful functions and calculating their FT
17 (10/26)	Ma	3. Convolution, correlations, the central limit theorem
18 (10/29)	Ma	4. LTI systems, more on convolution, FT as a linear system, what's special about $\exp(-i2\pi xy)$?
19 (11/2)	Ma	5. Convolution theorem and other Fourier theorems
20 (11/5)	Ma	6. The δ-function, distributions/generalized functions, re-defining FT

21 (11/9)	Ma	7. Discrete Fourier Transform, cyclic convolution
22 (11/12)	Ma	8. The Fast Fourier Transform algorithm
23 (11/16)	Ma	9. The Sampling Theorem, ghost function, and Nyquist frequency
24 (11/19)	Ma	10. Drawbacks of FT, wavelets, and continuous wavelet transforms, wavelet transform as a cross-correlation and constant-Q filtering
25 (11/26)	Ma	11. Discrete wavelet transforms and orthogonal wavelet decomposition, multi-resolution analysis
26 (11/30)	Ma	12. more on MRA, scaling function, digital filtering
27 (12/3)	Ma	13. Haar wavelets, filters, filter banks, multistage filter banks, and perfect reconstruction
28 (12/7)	Ma	14. Medical physics applications of Fourier and wavelet transforms

Grading:

Homework: 40%; Exams: 60%

Major references:

[1] Jorge Nocedal and Stephen J. Wright. *Numerical optimization*, 2nd edition, Springer Verlag, 1999.

[2] Ronald Bracewell, *The Fourier transform and its applications*, 3rd edition, McGraw Hill, 2000.

Additional references:

[1] Michael Greeberg, *Foundations of Applied Mathematics*, Prentice Hall, 1978

[2] E. Kreyszig, *Introductory functional analysis with applications*, volume 21. Wiley, 1989

Holidays: Labor Day 9/3/2018; Thanksgiving 11/22-23/2018.