

GS02 1052 – Imaging Science

Course description: This course provides a concise and coherent review of some commonly-encountered topics in applied mathematics, with a particular emphasis on their applications and relevance to imaging science. The course covers and is equally divided into two major sections: (i) image analysis methods and (ii) Fourier and wavelet transforms. Our image analysis approach provides a formalism for image registration, image reconstruction, image segmentation, and machine learning.

Learning objectives:

- Develop a mathematical formalism for approaching some commonly encountered problems in medical physics
- Interpret multiple distance measures for identifying accurate solutions
- Apply the mechanics of a neural network for image segmentation
- Understand common computational techniques for solving optimization problems inherent to image registration and image reconstruction
- Understand the mathematical formulation, properties, and limitations of Fourier transforms (continuous and discrete) and wavelet transforms and how they relate to some medical physics problems

Semester Begins: Monday, Aug. 30, 2021

Time and location: 11:00am-12:00pm (Tuesday, Friday) via zoom

Syllabus:

| Class | Instructor | Topic |
|------------|------------|--|
| 1 (8/31) | Fuentes | 1. Preliminaries - Algorithm Complexity, BLAS |
| 2 (9/3) | Fuentes | 2. Vector and metric spaces |
| 3 (9/7) | Fuentes | 3. Mutual Information |
| 4 (9/10) | Fuentes | 4. Entropy |
| 5 (9/14) | Fuentes | 5. Linear Independence, Equivalence of Norms |
| 6 (9/17) | Fuentes | 6. Linear Operators, Convolution |
| 7 (9/21) | Fuentes | 7. Image Segmentation |
| 8 (9/24) | Fuentes | 8. Analysis of Neural Network Structure |
| 9 (9/28) | Fuentes | 9. Rank and Nullity, Bounded Operator, Stability of Linear Systems |
| 10 (10/1) | Fuentes | 10. Inner Product, Orthogonality, Eigen-formulation |
| 11 (10/5) | Fuentes | 11. Optimization Characterization of Solution |
| 12 (10/8) | Fuentes | 12. Line search, Newton-CG Trust-Region Methods |
| 13 (10/12) | Fuentes | 13. Least Square, QR Decomposition |

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| 14 (10/15) | Fuentes | 14. L1 minimization |
| 15 (10/19) | Fuentes | 15. exam on optimization |
| 16 (10/22) | Mirkovic | 1. Where Nature does or calls for FT, definition of FT, notations, symmetry and FT, interpretation of FT |
| 17 (10/26) | Mirkovic | 2. Some useful functions and calculating their FT |
| 18 (10/29) | Mirkovic | 3. Convolution, correlations, the central limit theorem |
| 19 (11/2) | Mirkovic | 4. LTI systems, more on convolution, FT as a linear system, what's special about $\exp(-i2\pi xy)$? |
| 20 (11/5) | Mirkovic | 5. Convolution theorem and other Fourier theorems |
| 21 (11/9) | Mirkovic | 6. The δ -function, distributions/generalized functions, re-defining FT |
| 22 (11/12) | Mirkovic | 7. Discrete Fourier Transform, cyclic convolution |
| 23 (11/16) | Mirkovic | 8. The Fast Fourier Transform algorithm |
| 24 (11/19) | Mirkovic | 9. The Sampling Theorem, ghost function, and Nyquist frequency |
| 25 (11/23) | Mirkovic | 10. Drawbacks of FT, wavelets, and continuous wavelet transforms, wavelet transform as a cross-correlation and constant-Q filtering |
| 26 (11/30) | Mirkovic | 11. Discrete wavelet transforms and orthogonal wavelet decomposition, multi-resolution analysis |
| 27 (12/3) | Mirkovic | 12. more on MRA, scaling function, digital filtering |
| 28 (12/7) | Mirkovic | 13. Haar wavelets, filters, filter banks, multistage filter banks, and perfect reconstruction |
| 29 (12/10) | Mirkovic | 14. Medical physics applications of Fourier and wavelet transforms |
| 30 (TBD) | Mirkovic | 15. Exam on transforms |

Grading:

Homework: 40%; Exams (in-class): 60%

Major references:

[1] Jorge Nocedal and Stephen J. Wright. *Numerical optimization*, 2nd edition, Springer Verlag, 1999.

[2] Ronald Bracewell, *The Fourier transform and its applications*, 3rd edition, McGraw Hill, 2000.

Additional references:

[1] Michael Greeberg, *Foundations of Applied Mathematics*, Prentice Hall, 1978

[2] E. Kreyszig, *Introductory functional analysis with applications*, volume 21. Wiley, 1989

Holidays: Labor Day 9/6/2021; Thanksgiving 11/25-26/2021